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TRANSIENT THERMAL RESPONSE OF ENCLOSURES: THE INTEGRATED THERMAL TIME-CONSTANT

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Abstract—A method has been developed for theoretical computation of the integrated thermal time-constant of enclosures, hitherto unknown. The enclosure may consist of any number of different types of composite walls, roof and floor, whether all exposed or some unexposed. It may contain any other internal mass such as furniture. The enclosure may be ventilated as well. It is also possible to take into account the indoor radiation exchanges between the internal surfaces of the enclosure separately. This makes easy, the assessment of the thermal characteristics of the entire enclosure in terms of a single parameter.

NOMENCLATURE

A_s,	surface area perpendicular to the heat flow path [m²];
C',	volumetric heat capacity of air [kcal/m³ degC];
F_a,	shape factor;
F_e,	emissivity factor;
N,	number of air changes per hour [1/h];
$1/R$,	conductance or, surface heat transfer coefficient [kcal/m² h degC];
V,	volume [m³];
W,	internal heat source [kcal/h];
Y_{a0},	capacitative admittance of indoor air [kcal/h degC];
$\bar{Y}_c$,	average convective admittance for the internal surfaces [kcal/h degC];
$\bar{Y}'_c$,	average surface admittance for the internal surfaces combining the effects of radiative and convective exchanges [kcal/h degC];
$\bar{Y}_r$,	average radiative admittance for the internal surfaces [kcal/h degC];
e_1, e_2,	emissivity of internal surfaces;
h_c,	convective film heat-transfer conductance [kcal/m² h degC];
k,	thermal conductivity [kcal/m h degC];
l,	thickness [m];
p,	Laplace transform parameter;
t,	temperature [°C];
x,	space co-ordinate measured along the width of the wall fabric;
A', B', K_i, m, n, L, E, F, G, H,	constants defined in the text;
A, B, C,	elements of the wall transmission matrix defined in the text;
a,	thermal diffusivity [m²/h];
σ,	Stefan–Boltzmann Constant [kcal/m² h degK⁴];
τ,	time [h];
ψ_i,	roots of polynomials occurring in the text;
$\phi_R(\tau)$,	temperature response function of the indoor air due to unit step change of outdoor temperature [degC];
Z_0,	$= 1/k\beta$;
β,	$= \sqrt{(p/a)}$.

INTRODUCTION

THE IMPORTANCE of the concept of thermal time-constant of a building fabric in assessing its thermal characteristics, particularly when the fabric is exposed to conditions of unsteady heat flow, has already been brought out by the author [1, 2, 3]. While studying the problems of cooling of heated buildings, Bruckmayer [4] used this concept of thermal time-constant—a simple ratio of steady-state terms, for determining the thermal characteristics of individual building elements. Recently, Pratt and Ball [5] has shown from exact analytical considerations that both for homogeneous and multilayer composite building elements, Bruckmayer's thermal time-constant explains almost accurately, the transient heat-flow phenomenon under some simplified boundary conditions. Very recently, Warsi and Choudhury [6] have also worked out thermal time-constant values of a few two and three layered building elements from considerations of transient thermal reponse of buildings.

All these above investigations were directed towards finding the thermal time-constant values of individual building elements forming a wall of an enclosure having all the walls of similar construction and all exposed to identical excitations. It has been experimentally found [3] that there exist substantial indoor thermal radiative exchanges between the various elements of an enclosure. Moreover, buildings usually have roofs, walls and floors of different construction. As such, what was precisely needed was an integrated thermal time-constant of the entire enclosure. This integrated thermal time-constant should cover any complex structure of different types of homogeneous or multilayer constructions, the effect of various aspects such as the introduction of internal mass and ventilation and the effect of inclusion of the indoor inter-surface radiation exchanges. This paper presents a method for the computation of an integrated thermal time-constant of enclosures including the above variables by first obtaining theoretically the transfer function of the systems.

THEORETICAL

Wall transmission matrix

The one dimensional heat-conduction equation for a homogeneous rectangular wall fabric of uniform thickness and with the assumption that the heat losses at the edges are negligible, is

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau} \tag{1}$$

where the temperature, $t = t(x, \tau)$ is a function of the space co-ordinate, x and time τ.

The initial and boundary conditions are,

$$\begin{aligned} t &= 0 \quad \text{at} \quad \tau = 0 \quad \text{and} \quad 0 \leqslant x \leqslant l, \\ t &= t_0 \quad \text{at} \quad \tau > 0 \quad \text{and} \quad x = 0, \\ t &= t_l \quad \text{at} \quad \tau > 0 \quad \text{and} \quad x = l. \end{aligned}$$

Representing the Laplace transform of the various quantities mentioned above by the corresponding capital letters, such as,

$$L\, t(x, \tau) = \int_0^{\alpha} t(x, \tau)\, e^{-p\tau}\, d\tau \equiv T(x, p),$$

equation (1) is transformed into

$$\frac{\partial^2}{\partial x^2} T(x, p) = (p/\alpha)\, T(x, p). \tag{2}$$

The solution of equation (2) will be as

$$T(x, p) = A_1'\, e^{\beta x} + A_2'\, e^{-\beta x} \tag{3}$$

where $\beta = \sqrt{(p/\alpha)}$

With the help of the initial and boundary conditions and considering the following basic relationship

$$-k \frac{\partial}{\partial x} T(x, p) = H(x, p) \tag{4}$$

where $H(x, p)$ is the Laplace transform of heat flow at any point x, and further denoting

$$\begin{aligned} H &= H_0 \quad \text{at} \quad x = 0 \quad \text{and} \quad \tau > 0 \\ H &= H_l \quad \text{at} \quad x = l \quad \text{and} \quad \tau > 0 \end{aligned}$$

the constants (A_1' and A_2') may be determined and the solutions for T_l and H_l are obtained as follows.

$$T_l = T_0 \cosh \beta l - H_0 Z_0 \sinh \beta l \tag{5}$$

$$H_l = H_0 \cosh \beta l - (T_0/Z_0) \sinh \beta l \tag{6}$$

where $Z_0 = 1/k\beta$.

Equations (5) and (6) may be represented in the matrix form as,

$$\begin{bmatrix} T_l \\ H_l \end{bmatrix} = \begin{bmatrix} \cosh \beta l & -Z_0 \sinh \beta l \\ -(1/Z_0) \sinh \beta l & \cosh \beta l \end{bmatrix} . \begin{bmatrix} T_0 \\ H_0 \end{bmatrix} \tag{7}$$

On matrix inversion the following is obtained,

$$\begin{bmatrix} T_0 \\ H_0 \end{bmatrix} = \begin{bmatrix} \cosh \beta l & Z_0 \sinh \beta l \\ (1/Z_0) \sinh \beta l & \cosh \beta l \end{bmatrix} . \begin{bmatrix} T_l \\ H_l \end{bmatrix} \tag{8}$$

The square matrix containing the hyperbolic functions in the above equation (8) is called the wall transmission matrix. For a fabric having negligible heat capacity (such as a stagnant air layer), the transmission matrix reduces to

$$\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \tag{9}$$

For a composite wall having multilayered fabrics in series, the overall wall transmission matrix can be obtained by multiplying the individual transmission matrices of the fabrics in various layers following regular order along the positive direction of the x-axis as follows:

$$\begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & A_1 \end{bmatrix} . \begin{bmatrix} A_2 & B_2 \\ C_2 & A_2 \end{bmatrix} . \begin{bmatrix} A_3 & B_3 \\ C_3 & A_3 \end{bmatrix} \tag{10}$$

where the square matrix in the left-hand side of equation (10) represents the overall transmission matrix of the composite wall whereas the square matrices on the right-hand side represent the individual transmission matrices of the fabrics in the various layers along the positive direction of the x-axis. The elements A, B and C represent the corresponding elements of the square matrix in equation (8). Usually the contact resistances between the various layers are neglected.

If the fabric is moist and it is assumed that the moisture is distributed along the width of the fabric in a regular manner, then α and k of the material will be a function of the space coordinate, x, the nature of which could be determined from empirical relationships. In that case, the fabric may be considered to be constituted of different layers having constant α and k, in analogy to the lumped systems.

In equation (8), T_0, H_0 and T_l, H_l represent the temperatures and heat flows at the respective surfaces. If it is required to consider the temperatures of the fluid (ambient air or sol-air temperature etc.) in contact with a surface, the fluid film should be taken as a layer of zero heat capacity and the square matrix (9) should be used in the respective locations. In that case, the overall wall transmission matrix will include the surface films.

For operational conveniences, the elements of the transmission matrix (8) can be expanded in polynomials of p as follows:

$$\cosh \beta l = 1 + (l^2/2\alpha)\, p + (1/6)\,(l^2/2\alpha)^2 p^2 + (1/90)\,(l^2/2\alpha)^3 p^3 + \ldots \tag{11}$$

$$Z_0 \sinh \beta l = (l/k)\,[1 + (1/3)\,(l^2/2\alpha)\, p + (1/30)\,(l^2/2\alpha)^2 p^2 + (1/630)\,(l^2/2\alpha)^3 p^3 + \ldots] \tag{12}$$

$$(1/Z_0) \sinh \beta l = (2k/l)\,[(l^2/2\alpha)\, p + (1/3)\,(l^2/2\alpha)^2 p^2 + (1/30)\,(l^2/2\alpha)^3 p^3 + \ldots] \tag{13}$$

It is thus possible to express each element of the wall transmission matrix in terms of a polynomial in p. The degree of the polynomial in each case will be m times the number of layers in the multilayered panels; m being the highest degree of p taken in the above expansions (11–13). The value of m will be dependent on the number of terms that would be required in the above expansions to attain a certain degree of accuracy. Expansions of similar nature have also been mentioned by Stephenson [7].

Thermal-circuit representation of an enclosure

The mechanism of heat transmission and various heat exchanges in an enclosure can be very well represented by a thermal circuit originally due to Nottage and Parmele [8], and whose application in the prediction of indoor climate has been reviewed by the author [9]. The heat is transmitted indoors through the exposed walls (including roofs and floors, if applicable) of the enclosure whose surface areas are large compared to the thickness. The heat flow paths through the exposed walls can thus be treated as passive quadripoles. For unexposed internal walls, the mid-plane (at half the thickness) can be taken as adiabatic planes and as such, these also can be treated as passive quadripoles. The assumptions to be made are as follows: (a) the walls, whether exposed or unexposed, have isothermal surfaces indicating one dimensional heat flow; (b) the air in the enclosure has uniform temperature; (c) the thermophysical properties of materials and of the surfaces are constant and time-invariant.

Thus equation (10) for the overall wall

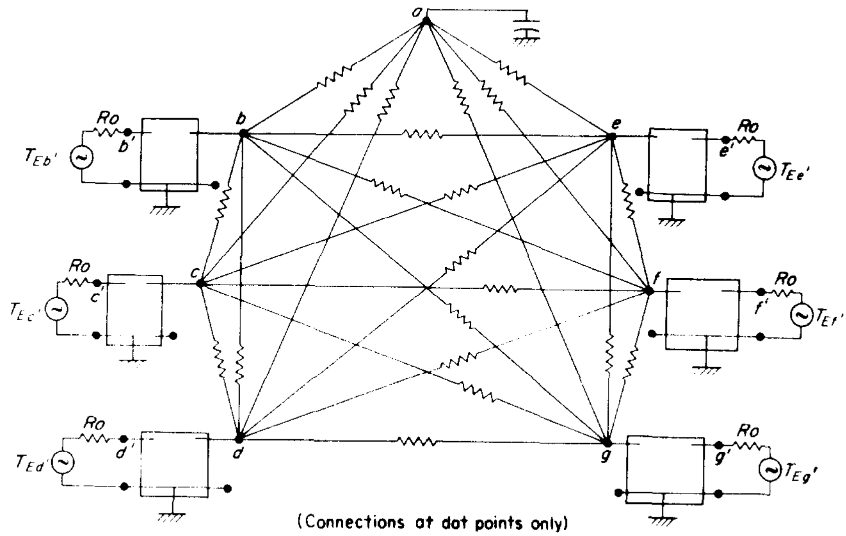

FIG. 1. Thermal circuit diagram using quadripoles for conduction paths of an enclosure.

transmission matrix could be used here for the conduction flow through the fabrics. The radiative heat exchanges between the various surfaces (i, j) and the convective exchanges from these surfaces (i or j) to the indoor air (a) are represented by radiative and convective admittances defined by

$$\text{Convective admittance, } Y_{ia} = h_c(A_s)_i$$

$$\text{Radiative admittance, } Y_{ij} = F_e\, F_a(A_s)_i\, 4\sigma(t_{\text{mean}} + 273{\cdot}16)^3$$

where $F_e = e_1 . e_2$.

The capacitative admittance of the indoor air is defined by

$$Y_{a0} = VC'p.$$

Let the enclosure (having six enclosing walls for simplicity) be represented by a thermal-circuit, Fig. 1, where each box represents a wall of the enclosure (including roof and floor) and is being treated as a passive quadripole. Let the indoor air temperature be represented by T_a and the temperatures and heat flows at the various surfaces be represented by T and H with proper subscripts in the transformed plane. Let T_E with the corresponding dashed subscripts represent the external air or sol-air temperature.

The energy equation which is to be considered at each surface nodes, say b, of the thermal-circuit (Fig. 1) is

$$H_b = Y_{ba}(T_b - T_a) + \sum_{j=1}^{n-1} Y_{bj}(T_b - T_j) \quad (14)$$

where j refers to other $(n - 1)$ internal surfaces in the enclosure. The right-hand side of the equation (14) is to be equated with

$$[(T_{Eb'}/F) - (T_b\, E/F)]$$

for exposed elements and with $(-T_bG/H)$ for internal unexposed elements. $T_{Eb'}$ is the corresponding external sol-air or air temperature, or in brief the excitation temperature. E, F, G, and H, the elements of the wall transmission matrices, should include the film air layer for the outer surfaces (equation 10).

Further the energy equation to be considered at the node a representing indoor air, is

$$\sum_{j=1}^{n} (A_s)_j\, Y_{ja}(T_j - T_a) + W = 0 \quad (15)$$

where W, the internal heat source in the enclosure, may exist due to occupants, lighting, ventilation (sources or sinks), or direct solar irradiation through transparent openings; W may be constant or time dependent.

Thus we get $(n + 1)$ simultaneous linear equations in terms of temperature, modified admittances (including transmission matrices for the fabrics), internal heat sources etc., and where, there are n isothermal surfaces (whether exposed or unexposed) in the enclosure.

If further, it is assumed that all the external surfaces of the enclosure are exposed to the same excitation temperature, T_E and W is only due to the ventilation of the outdoor air, the following expression is obtained:

$$\frac{T_a}{T_E} = \frac{\begin{vmatrix} -NVC' & Y_{ab} & Y_{ac} & Y_{ad} & Y_{ae} & Y_{af} & Y_{ag} \\ -(A_s/F)_b & -\Sigma Y_b' & Y_{bc} & Y_{bd} & Y_{be} & Y_{bf} & Y_{bg} \\ -(A_s/F)_c & Y_{cb} & -\Sigma Y_c' & Y_{cd} & Y_{ce} & Y_{cf} & Y_{cg} \\ -(A_s/F)_d & Y_{db} & Y_{dc} & -\Sigma Y_d' & Y_{de} & Y_{df} & Y_{dg} \\ -(A_s/F)_e & Y_{eb} & Y_{ec} & Y_{ed} & -\Sigma Y_e' & Y_{ef} & Y_{eg} \\ -(A_s/F)_f & Y_{fb} & Y_{fc} & Y_{fd} & Y_{fe} & -\Sigma Y_f' & Y_{fg} \\ -(A_s/F)_g & Y_{gb} & Y_{gc} & Y_{gd} & Y_{ge} & Y_{gf} & -\Sigma Y_g' \end{vmatrix}}{\begin{vmatrix} -\Sigma Y_a' & Y_{ab} & Y_{ac} & Y_{ad} & Y_{ae} & Y_{af} & Y_{ag} \\ Y_{ba} & -\Sigma Y_b' & Y_{bc} & Y_{bd} & Y_{be} & Y_{bf} & Y_{bg} \\ Y_{ca} & Y_{cb} & -\Sigma Y_c' & Y_{cd} & Y_{ce} & Y_{cf} & Y_{cg} \\ Y_{da} & Y_{db} & Y_{dc} & -\Sigma Y_d' & Y_{de} & Y_{df} & Y_{df} \\ Y_{ea} & Y_{eb} & Y_{ec} & Y_{ed} & -\Sigma Y_e' & Y_{ef} & Y_{eg} \\ Y_{fa} & Y_{fb} & Y_{fc} & Y_{fd} & Y_{fe} & -\Sigma Y_f' & Y_{fg} \\ Y_{ga} & Y_{gb} & Y_{gc} & Y_{gd} & Y_{ge} & Y_{gf} & -\Sigma Y_g' \end{vmatrix}} \tag{16}$$

where $\Sigma Y_j' = (A_s E/F)_j + Y_{ja} + \Sigma Y_{ji}$

and $\Sigma Y_a' = Y_{a0} + NVC' + \Sigma Y_{aj}$

For an unexposed internal wall (say i) such as partition wall (including furnitures) in an enclosure, the expression $-(A_s/F)_i$ in the first column of the numerator in the above equation (16) should be treated as zero. Further, the expression to be taken for $\Sigma Y_i'$ in both the numerator and the denominator (equation 16) is

$$\Sigma Y_i' = (A_s G/H)_i + Y_{ia} + \Sigma Y_{ij}$$

Transfer function

Let a time-invariant linear system be considered having one or more driving functions and one or more response functions. Let $r = r(\tau)$ be a particular response function and $d = d(\tau)$ be the particular driving function. Let all other driving functions be set to zero. Let the system have no residual effect from any earlier driving excitations. Let $R(p)$ be the Laplace transform of the normal response and $D(p)$ be the Laplace transform of the single driving function causing the response. Then let

$$TF(p) = R(p)/D(p) \tag{17}$$

where $TF(p)$ is called the transfer function of the system with respect to $r(\tau)$ and $d(\tau)$ but which is independent of $D(p)$.

In the equation (16), both T_a and T_E are the Laplace transforms of the response and excitation temperature functions respectively. Hence, the ratio of the two, i.e. equation (16) is the transfer function of the entire enclosure which includes the integrated effect of the intersurface radiation coupling, the surface–air convection coupling. It also includes the effect of introduction of internal mass and ventilation in addition to the transmission and storage characteristics of wall fabrics of homogeneous or composite construction consisting of any number of layers.

With the simplifying assumptions, that the convective and radiative admittances for all the internal surfaces of exposed or unexposed elements in the enclosure are similar and equal to $\bar{Y}_c$ and $\bar{Y}_r$ respectively (i.e. say an enclosure taken as a cube having similar surface finishes), the resulting transfer function will be,

$$TF(p) = \frac{\sum_{j=1}^{n}\left[(A_s/F)_j \frac{\bar{Y}_c}{\Sigma Y'_j + \bar{Y}_r}\right] + NVC'\left[\frac{\Sigma Y'_b}{\Sigma Y'_b + \bar{Y}_r} - \sum_{j=1}^{n-1}\frac{\bar{Y}_r}{\Sigma Y'_j + \bar{Y}_r}\right]}{(\Sigma Y'_a)\left[\frac{\Sigma Y'_b}{\Sigma Y'_b + \bar{Y}_r} - \sum_{j=1}^{n-1}\frac{\bar{Y}_r}{\Sigma Y'_j + \bar{Y}_r}\right] - \sum_{j=1}^{n}\frac{(\bar{Y}_c)^2}{\Sigma Y'_j + \bar{Y}_r}} \tag{18}$$

If in equation (19), the thermal capacity of the indoor air, which is comparatively very small, is neglected, the resulting expression for the transfer function would be the same as that which could be obtained starting with the equations used by Muncey [10].

Let us consider a very simple case of an unventilated enclosure (a cube) having no internal mass and with all the six walls of similar construction. When such an enclosure is exposed to a single excitation on all the external surfaces, it would not be unrealistic to consider that the mean radiant temperature of the indoor environment and its ambient air temperature are similar. Hence, the assumption of the surface admittance ($\bar{Y}'_c$) is reasonable. The transfer function of this enclosure is then given by

$$TF(p) = \frac{1}{\frac{1}{6}Y_{a0}\left[\frac{E}{\bar{Y}'_c} + \frac{F}{A_s}\right] + E} \tag{20}$$

If further, Y_{a0} is neglected the transfer function is reduced to $(1/E)$.

Further, if the indoor intersurface radiation exchanges are neglected as a separate entity (i.e. $\bar{Y}_r = Y_{ij} = Y_{ji} = 0$), the transfer function gets simplified to

$$TF(p) = \frac{\sum_{j=1}^{n}\left[(A_s/F)_j \frac{\bar{Y}'_c}{\Sigma Y'_j}\right] + NVC'}{\Sigma Y'_a - \sum_{j=1}^{n}\frac{(\bar{Y}'_c)^2}{\Sigma Y'_j}} \tag{19}$$

The use of the surface admittance $\bar{Y}'_c$ assumes that the mean radiant temperature of the indoor environment and the indoor air temperatures are similar—an assumption which may not be always true. In both the expressions (18 and 19), internal mass, if any, is to be treated as stated earlier. It should however be noted that in equation (18), any element other than the element b is to be treated as internal mass. Further, equation (19) is applicable to an enclosure, which is not necessarily a cube.

In the above enclosure, if one of the walls is treated as an internal (partition wall) element, and the use of $\bar{Y}'_c$ is retained, the transfer function of the enclosure will be

$$TF(p) = \frac{5\,A_s\bar{Y}'_c\,(A_sG + \bar{Y}'_cH)}{(Y_{a0} + 6\bar{Y}'_c)\,(A_sE + \bar{Y}'_cF)\,(A_sG + \bar{Y}'_cH) - (\bar{Y}'_c)^2\,[5F\,(A_sG + \bar{Y}'_cH) + H\,(A_sE + \bar{Y}'_cF)]} \tag{21}$$

In general, the expression of the transfer function, $TF(p)$, may be written as a ratio of two polynomials in p. For unventilated enclosures ($N = 0$), the degree of the polynomial in the denominator will be $(mL + 1)$ where m is the highest degree of p taken for the expansions of the elements of the transmission matrices, (equations 11–13) and $L = L_0 + L_i$; L_0 and L_i being the total number of layers in different types of exposed and internal (unexposed) walls respectively. The term walls, however, includes roofs, floors, and also furniture. If in the enclosure, more than one of the exposed walls are of identical construction, only one of these walls is to be considered for determining the value of L_0. The internal unexposed walls should be treated similarly. The degree of the polynomial in the numerator will be less than $(mL + 1)$, the extent of which will be dependent on the number of identical exposed or unexposed walls. But if in addition, the enclosure is ventilated $(N \neq 0)$, the degree of the polynomial in both the numerator and the denominator will be the same $(mL + 1)$. It may be pointed out that the inclusion of ventilation or internal radiative admittances do not alter the degree of the polynomials themselves. If the thermal capacity of the indoor air is neglected, the degree of the polynomials is reduced by one. The coefficients of the two polynomials can be obtained from equation (16). These are dependent on the thickness, thermal diffusivity and thermal resistances of the fabrics (assumed homogeneous) in the individual layers of the multilayer walls and their surface areas. These are also dependent upon the surface conductances of the outer surfaces and upon the surface admittances or the radiative and convective admittances for the internal surfaces.

Thermal time-constant

Once the integrated transfer function $TF(p)$ of an enclosure is determined as detailed above, the response of the enclosure for any given external excitation or forcing function may be obtained by first multiplying the $TF(p)$ with the Laplace transform of the excitation function and thereafter obtaining the inverse Laplace transform of the end product, using partial fraction expansion.

In the present work, a "unit step function" has been used as a forcing function. The temperature response function $[\phi_r(\tau)]$ of the indoor air in the enclosure due to this unit step external excitation is

$$\phi_r(\tau) = 1 - \sum_{i=1}^{m} K_i \exp(-\psi_i \tau) \qquad (22)$$

where ψ_i are the roots of the polynomial in p in the denominator of the $1/p$ multiplied transfer function, and K_i are the constants derived in the partial fraction expansions required as above. The time required by the response function $[\phi_r(\tau)]$ of the enclosure after the external excitation has been applied, to attain 63·21 per cent $(= 1 - 1/e$, where e is the base of natural logarithm) of the steady state value is defined as the thermal time-constant of the enclosure.

It is seen that the main work of computation is to obtain the partial fractions of the ratio of the two resulting polynomials. This in turn would mean the determination of the roots of $(mL + 1)$ degree polynomial in the denominator. This has been done by Muller's method [11] using a digital computor.

Thus, once the integrated thermal time-constants of enclosures are worked out, an accurate assessment of the comparative thermal characteristics of various enclosures subjected to any climatic stress is possible by one simple parameter, which was not feasible earlier.

NUMERICAL EXAMPLES AND DISCUSSION

Some simple cases have been worked out to illustrate the use of the method developed above. It should be noted that the solution for the response function, as obtained (equation 22) would be strictly true for large times. Since the thermal time-constant of any fabric is always greater than its $(l^2/2\alpha)$ and considering the numerical values of $(l^2/2\alpha)$ for the range of materials used in buildings, an approximation in the expansions of the wall transmission matrix elements (equations 11–13) by neglecting the terms containing powers higher than 2 of $(l^2/2\alpha)$ is expected to give thermal time-constant values of an accuracy which should be sufficient for most of the engineering applications. In the following numerical examples, a value of $m = 2$ has, therefore, been taken.

The thermophysical properties of the different materials and surfaces are given in Table 1. The details of the enclosures with the various boundary conditions together with the integrated thermal time-constant values are presented in Table 2. The thermal time-constant values have been obtained by solving the transcendental equations $[\phi_r(\tau)]$ for 63·21 per cent response by

Table 1. Thermophysical properties of materials and surfaces

	Materials: Brickwork	Materials: Cement concrete
	$k = 0{\cdot}833$ kcal/m h degC $a = 2{\cdot}36 \times 10^{-3}$ m²/h	$k = 1{\cdot}711$ kcal/m h degC $a = 3{\cdot}35 \times 10^{-3}$ m²/h
	Air $C' = 0{\cdot}2806$ kcal/m³ degC	
Surfaces		
$1/R_0$	$= 19{\cdot}5315$ kcal/m²h degC (outer surface heat-transfer coefficient)	
$\bar{Y}_c$	$= 3{\cdot}4177\ A_s$ kcal/h degC	
$\bar{Y}_r$	$= 0{\cdot}9277\ A_s$ kcal/h degC (for an average temperature of 29·5°C)	
F_a	$= 0{\cdot}2$, both for adjacent and opposite planes [12]	
F_e	$= (0{\cdot}92)^2 = 0{\cdot}85$	
$\bar{Y}_c'$	$= \bar{Y}_c + A_s F_e 4\sigma(t_{\text{mean}} + 273{\cdot}16)^3$, (since $F_a = 1$) $= 8{\cdot}056\ A_s$ kcal/h degC (for an average temperature of 29·5°C)	

Newton's method using a digital computor. These may also be obtained graphically.

Thermal time-constant values of 22·86 cm thick and 11·43 cm thick brick masonry walls (case Nos. 1 and 2) as computed by Bruckmayer's [4] steady-state method are also included in parenthesis in Table 2, for comparison. As stated earlier, it has been reported [5], based on exact mathematical analysis that the thermal time-constant as obtained from the transient response of a sealed enclosure with all the six surfaces identical and all exposed to the same external excitation and further containing no internal mass is almost identical to the time-constant of a similar element obtained from the above mentioned steady-state method. It is seen (Table 2) that the time-constant values derived by the present method under the above idealized conditions (case Nos. 1 and 2) are within two per cent of those obtained by Bruckmayer's steady-state method; however this error becomes negligible when in the present method of computation the values of m are taken as 3, 4, 5 or more. Hence the use of the assumption of $m = 2$ for most of the engineering applications is justifiable as the maximum error is within two per cent only. It is therefore expected that under generalized boundary conditions, this method will yield results of comparable accuracy.

It is clearly seen (Table 2) that the increase of internal mass increases the integrated thermal time-constant of an enclosure (case Nos. 3 and 4).

The neglect of the thermal capacity of the indoor air mass as has been done by Muncey [10] and Warsi and Choudhury [6] causes an insignificant error in the time-constant value of the order of 0·7 per cent in very small enclosures, 0·305 m cube (say a small edge insulated thermal model). The magnitude of the error is increased to the order of 1·6 per cent for regular rooms (3·05 m cube). This error is also small but for larger halls (30·5 m cube) it is of the order of 8·8 per cent which is not negligible (case Nos. 4, 6, 7, and 8).

In order to include the effect of indoor inter-surface radiation exchanges, the radiative and convective admittances of the indoor surfaces have been considered individually in case No. 5. It is seen that (case Nos. 4 and 5) the effect of this treatment is only of the order of 1·6 per cent on the time-constant values. This is perhaps due to the simplified assumptions made for the constancy of various admittances, particularly of the convective surface admittances irrespective of orientation. This may also be attributed to the conventional procedure adopted for combining the effects of radiative and convective admittances to form an overall surface admittance assuming the mean radiant temperature of the indoor environment similar to its ambient air temperature. In order to correctly assess the

Table 2. Integrated thermal time constants of various enclosures under different boundary conditions

Case No.	Details of enclosure (cubical)	Internal volume/ exposed wall area (m)	Internal mass per unit area of exposed wall (kcal/ m^2 degC)	Internal air mass per unit area of exposed wall (kcal/ m^2 degC)	Ventilation, number of air changes per hour (1/h)	Integrated thermal time-constant of the enclosure (h)
1.	22·86 cm thick brick masonry enclosure with all the six walls exposed	—	nil	nil	nil	15·5 (15·2)
2.	11·43 cm thick brick masonry enclosure with all the six walls exposed	—	nil	nil	nil	4·9 (4·8)
3.	An enclosure having 11·43 cm thick brick masonry for the five exposed walls. Sixth wall of 5·08 cm thick dense cement concrete treated as an internal partition wall	—	2·596	nil	nil	5·72
4.	An enclosure having 11·43 cm thick brick masonry for the five exposed walls. Sixth wall of 30·48 cm thick dense concrete treated as an internal floor	—	15·576	nil	nil	7·02
5.*	Same as 4	—	15·576	nil	nil	7·13
6.	Same as 4	0·061	15·576	0·0171	nil	7·07
7.	Same as 4	0·61	15·576	0·171	nil	7·13
8.	Same as 4	6·1	15·576	1·71	nil	7·64
9.	Same as 4	0·061	15·576	nil	4	6·86
10.	Same as 4	0·61	15·576	nil	4	5·71
11.	Same as 4	6·1	15·576	nil	4	1·78

* In case No. 5 only, $\bar{Y}_c$ and $\bar{Y}_r$ have been used separately. In all other cases $\bar{Y}_c'$ has been used neglecting $\bar{Y}_r$ as an individual entity. In cases Nos. 1–5, the third col. has been left blank as the same has no effect on the last col. The time-constant values shown in parenthesis for case Nos. 1 and 2 are from steady-state method of Bruckmayer [4].

contribution of the indoor intersurface radiation exchanges, a detailed study towards the exact behaviour of the various surface admittances are essential prerequisites.

Ventilation has been introduced in three cases (case Nos. 9–11). It is seen that the ventilation causes a reduction of the time-constant value, which is a logical expectation. It is further observed, on comparison with case No. 4, that a particular air-change rate of ventilation ($N = 4$) reduces the time-constant values by 2·3 per cent for very small enclosures (0·305 m cube). For regular room dimensions (3·05 m cube), the reduction is 18·7 per cent and it is of the order of 74·6 per cent for large hall dimensions (30·5 m cube).

CONCLUSION

The method developed in this paper predicts theoretically the integrated thermal time-constant of an enclosure as a whole, which was not feasible earlier. This method is capable of taking into account, an enclosure with different types of walls, each composed of any number of layers; whether all those walls are exposed or some are unexposed. In addition, it takes into account, the introduction of any other internal mass and ventilation. It also includes the effect of indoor intersurface radiative exchanges in the enclosure. The method is comparatively simpler considering the large number of parameters, it can handle. Lastly, this method, which has been found to be capable of predicting precisely the integrated thermal time-constant of enclosures for simple boundary conditions is expected to yield reliable results for generalized boundary conditions including all the above variables, and which should be sufficiently accurate for most of the engineering applications.

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Résumé—Une méthode pour le calcul théorique de la constante de temps thermique intégrée des enceintes, grandeur inconnue jusqu'alors, a été exposée. L'enceinte peut consister en un nombre quelconque de murs composites de différents types, d'un plafond et d'un plancher, soit, tous exposés, soit, avec quelques parois non exposées. Elle peut contenir n'importe quelle masse interne telle que des meubles. L'enceinte peut être également ventilée. Il est possible aussi de tenir compte des échanges intérieurs de rayonnement entre chaque surface interne de l'enceinte. Ceci rend facile l'évaluation des caractéristiques thermiques de l'enceinte tout entière en fonction d'un paramètre unique.

Zusammenfassung—Für die theoretische Berechnung der bisher unbekannten gesamten, thermischen Zeitkonstante von Hohlräumen wurde ein Verfahren entwickelt. Der Hohlraum kann aus einer beliebigen Zahl von nach verschiedenen Arten zusammengesetzten Wänden mit Dach und Boden bestehen, wobei sich alle oder auch nur einige Wände instationär erwärmen. In ihm kann sich jede andere Masse wie z.B. Möbel befinden. Der Raum kann auch belüftet sein. Ebenso ist es möglich, den Strahlungsaustausch im Inneren zwischen den Wänden des Raumes getrennt mit einzubeziehen. Dies erleichtert die Abschätzung der thermischen Kenngrössen des ganzen Hohlraumes und ergibt Ausdrücke mit einem einzigen Parameter.

Аннотация—Разработан новый метод теоретического расчета интегральных временных констант ограждений. Ограждение может состоять из любого числа многослойных стен, пола и потолка различных типов, при чем все они могут быть подвержены воздействию наружного воздуха или не соприкасаться с ним. Оно может содержать любые внутренние массы, например, мебель. Ограждение может быть также вентилируемым.

Можно также учитывать лучистый теплообмен между внутренними поверхностями ограждения. Это облегчит нахождение тепловых характеристик всего ограждения с помощью одного параметра.